3. $y'' - 4y' + 13y = 0 \quad , y(0) = 2 \quad , y'(0) = 1$

$\lambda^2 - 4\lambda + 13 = 0$

$\lambda = 2 \pm 3i \quad \Rightarrow \quad y_1 = e^{(2+3i)x} \quad , y_2 = e^{(2-3i)x}$

$y = c_1 e^{(2+3i)x} + c_2 e^{(2-3i)x}$

$y(0) = c_1 + c_2 = 2 \quad \Rightarrow \quad c_2 = -c_1 + 2$

$y' = c_1(2+3i)e^{(2+3i)x} + c_2(2-3i)e^{(2-3i)x}$

$y'(0) = c_1(2+3i) + c_2(2-3i) = 1$

$c_1(2+3i) + (2-c_1)(2-3i) = 1$

$c_1 \cdot 6i + 4 - 6i = 1$

$6ic_1 = 6i - 3$

$c_1 = \frac{6i-3}{6i} = 1 + \frac{1}{2}i$

$c_2 = -(1 + \tfrac{1}{2}i) + 2 = 1 - \tfrac{1}{2}i$

$y = (1+\tfrac{1}{2}i)e^{(2+3i)x} + (1-\tfrac{1}{2}i)e^{(2-3i)x}$

$= (1+\tfrac{1}{2}i)(e^{2x})e^{3ix} + (1-\tfrac{1}{2}i)e^{2x}e^{-3ix}$ (use: $e^{i\theta} = \cos\theta + i\sin\theta$)

$= e^{2x}\left[(1+\tfrac{1}{2}i)(\cos 3x + i\sin 3x) + (1-\tfrac{1}{2}i)(\cos 3x - i\sin 3x)\right]$

$= e^{2x}\big[\cos 3x + i\sin 3x + \tfrac{1}{2}i\cos 3x - \tfrac{1}{2}\sin 3x$

$\quad + \cos 3x - i\sin 3x - \tfrac{1}{2}i\cos 3x - \tfrac{1}{2}\sin 3x\big]$

$= e^{2x}\left[2\cos 3x - \sin 3x\right]$

OR use the short cut way:

if $\lambda = a \pm bi \quad \Rightarrow \quad y = e^{ax}\left[c_1\cos bx + c_2\sin bx\right]$

note: these c_1 & c_2 are different from above

Everything here is in the Real numbers!

but that's ok b/c our answer will always be real anyway.